CONSTITUTIVE LAWS FOR SEA ICE DYNAMICS
MULTISCALE MICROMECHANICS OF ICE FIELDS

Dr. Martin Ostoja-Starzewski
3454 Sheridan Chase
Marietta, GA 30067
Martin Ostoja <104145.3726@compuserve.com>
martin_ostoja@hotmail.com
Voice: 770-578-9386 FAX: 770-971-5674
Award No. N0001496C0174

LONG TERM GOALS

To formulate constitutive laws and solution methods for ice fields viewed as heterogeneous multiscale materials from the standpoint of micromechanics of random media.

To develop a stochastic-computational model for mechanics of pack ice treated as a spatially random granular material of Mohr-Coulomb type.

APPROACH

A spatially random granular Mohr-Coulomb material is obtained from the classical, homogeneous continuum model by taking the internal friction angle ρ and the resistance in tension H as a vector random field. These two constitutive coefficients specify a meso-continuum approximation on a meso-scale corresponding to the actual choice of spacing of a finite difference net of characteristics (Fig. 1) employed in the solution of a given boundary value problem. The term 'microscale' connotes the scale of a single ice floe grain indicated in the figure below.

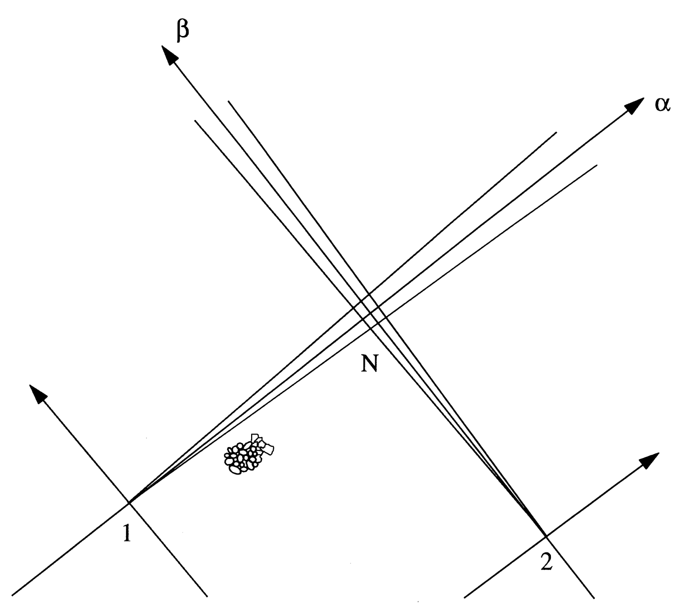

Fig. 1 Showing the mesoscale (spacing of slip-lines/characteristics) relative to the microscale (size of a single ice floe grain), and wedges of diffusing characteristics.

Report Documentation Page

Form Approved
OMB No. 0704-0188

1. REPORT DATE **30 SEP 1997**	2. REPORT TYPE	3. DATES COVERED **00-00-1997 to 00-00-1997**

4. TITLE AND SUBTITLE **Constitutive Laws for Sea Ice Dynamics Multiscale Micromechanics of Ice Fields**	5a. CONTRACT NUMBER 5b. GRANT NUMBER 5c. PROGRAM ELEMENT NUMBER
6. AUTHOR(S)	5d. PROJECT NUMBER 5e. TASK NUMBER 5f. WORK UNIT NUMBER
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES) **Dr. Martin Ostoja-Starzewski,3454 Sheridan Chase,Marietta,GA,30067**	8. PERFORMING ORGANIZATION REPORT NUMBER
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)	10. SPONSOR/MONITOR’S ACRONYM(S) 11. SPONSOR/MONITOR’S REPORT NUMBER(S)

12. DISTRIBUTION/AVAILABILITY STATEMENT
Approved for public release; distribution unlimited

13. SUPPLEMENTARY NOTES

14. ABSTRACT

15. SUBJECT TERMS

16. SECURITY CLASSIFICATION OF:			17. LIMITATION OF ABSTRACT	18. NUMBER OF PAGES	19a. NAME OF RESPONSIBLE PERSON
a. REPORT **unclassified**	b. ABSTRACT **unclassified**	c. THIS PAGE **unclassified**	**Same as Report (SAR)**	**4**	

ACCOMPLISHMENTS

Explicit equations have been derived for the determination of the alpha-family and the beta-family of characteristics in a Mohr-Coulomb medium. These are

$$d\sigma + 2\sigma \tan\rho\, d\varphi = \gamma(\tan\rho\, dx + dy) + \left(\sigma \frac{\partial \rho}{\partial y} - \frac{\partial H}{\partial x} + \frac{\partial H}{\partial x}\tan\rho\right)dx + \left(-\sigma \frac{\partial \rho}{\partial y} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial x}\tan\rho\right)dy$$

along the α family of characteristics given by

$$\frac{dy}{dx} = \tan(\varphi + \varepsilon)$$

and

$$d\sigma + 2\sigma \tan\rho\, d\varphi = \gamma(-\tan\rho\, dx + dy) + \left(-\sigma \frac{\partial \rho}{\partial y} - \frac{\partial H}{\partial x} - \frac{\partial H}{\partial x}\tan\rho\right)dx + \left(\sigma \frac{\partial \rho}{\partial y} + \frac{\partial H}{\partial y} - \frac{\partial H}{\partial x}\tan\rho\right)dy$$

along the β family of characteristics given by

$$\frac{dy}{dx} = \tan(\varphi - \varepsilon)$$

Here ε is an auxiliary angle equal to $\pi/4$ - $\rho/2$.

It is noteworthy that:

(i) This derivation generalizes the classical theory of Mohr-Coulomb plasticity (e.g., Nedderman, 1995; Sokolovskii, 1965; Szczepinski, 1974) to a situation of spatially inhomogeneous material properties.

(ii) This derivation is analogous, albeit more complicated, than an analogous derivation carried out for materials obeying the Huber-Mises yield condition (typically, metals); (Ostoja-Starzewski, 1992; Ostoja-Starzewski and Ilies, 1996).

Due to the spatial fluctuations of ρ and H in the granular ice field medium, the characteristics display perturbations in their evolution throughout the domain of dependence. Consequently, wedges of diffusing characteristics replace the unique unperturbed characteristics of the reference deterministic (homogeneous) medium problem, see Fig. 1. The reference medium is defined as the one in which ρ and H are constant. The entire formulation falls into a general framework of micromechancally based stochastic finite element/difference methods, where a single finite element (or difference) plays the role of a bridge between the small scale variability and the macroscopic response (Ostoja-Starzewski, 1993).

A computer program has been developed to investigate the sensitivity of solutions to various types of noises in ρ and H as compared to the reference medium. Assessment of the influence of spatial material randomness on the plastic response of granular-type ice fields for a range of these two key parameters has been carried out.

The study presents a possibility of incorporation of micromechanics into prediction models of ice field mechanics.

SCIENTIFIC/TECHNICAL RESULTS

Development of a technique for inclusion of small scale inhomogeneities in mechanics of ice fields has been the objective of this research project. The study has been set in the context of Mohr-Coulomb plastic behavior, but is extendable to other, more realistic constitutive models. The method allows to grasp the said small scale, random material variability in the solution of large scale boundary value problems. In particular, we propose to study boundary value problems in this field by a finite difference method based on the method of characteristics generalized to random media. This relies on solving a given stochastic boundary value problem directly by calculating a large number of realizations in a Monte Carlo sense. With the power of today's computers this presents no obstacle and yields (very rapidly) practically the whole range of possible behaviors; in fact, large problems can be treated on personal computers or work stations. Thus, the probability distributions of slip-line velocity and stress fields down to within the resolution of the finite difference net, i.e. the mesoscale can be obtained.

A comparison has also been made of the effects of small scale material variability on Mohr-Coulomb (MC) type versus Huber-Mises (HM) type plastic media. Following major conclusions can be drawn here:

i) For very weak noise there is only a small difference between the ensemble average net of slip-lines of the stochastic problem (i.e., for a random medium) and the net of a corresponding deterministic problem (i.e., for a homogeneous medium). This difference and the accompanying scatter increase as material parameter fluctuations grow - or, equivalently, as the mesoscale d decreases.

ii) Materials governed by the MC yield criterion appear to be more sensitive to noise than those governed by the HM criterion. In general, parameter ρ has a stronger influence than H.

iii) There is practically no difference between results of a forward or a backward integration method for the HM materials; but, by contrast, backward differencing is recommended for the MC media.

In view of the nonlinear and stochastic nature of the problem, the above results could not be obtained analytically, and the numerical analysis was the optimal way to proceed.

Finally, we mention that the method developed in this study can also be extended to the hardening behavior and anisotropic yield conditions. Indeed, anisotropic yield conditions at the meso-scale are expected from a micromechanical derivation; this subject presents one of the most important challenges for further research on the subject of multiscale mechanics of ice fields.

Another goal, which should be considered in future research and can be attmpted with a technique such as developed here, is the morphogenesis of multiscale, and often fractal, patterns in ice fields that are typically observed. This has first been considered by Ostoja-Starzewski (1990).

IMPACT FOR SCIENCE (and/or) SYSTEMS APPLICATIONS

This study demonstrates a method for treatment of multiscale mechanical phenoemena. A need for such a technique has been noted by Overland *et al* (1995). The analysis carried out also illustrates the importance of small scale (ice floe length scales) random variability in material properties on macroscale response.

RELATED PROJECTS

None during the award period of this project.

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